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கல்வி அமைச்சு
Ministry of Education

G.C.E.(A.L) Support Seminar - 2022

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

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மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  Answer ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  Answer ***five*** *questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

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Part A

1. Using the Principles of Mathematical Induction prove that $\sum_{r=1}^{n} 6r(r-1) = 2n(n^2-1)$ for all $n \in \mathbb{Z}^+$

2. Sketch the graphs of $y = |2x-2| - x$ and $y = 2|x-2| - 2x$ in the same figure. Hence or otherwise, find all real values of x satisfying the inequality $|2x-4| - 2|x-2| > x$

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3. On a sketch of an Argand diagram, shade the region whose point represent complex number z satisfying the inequality $|z-2-2i| \le 1$ and $rg(z-4i) \ge -\frac{\pi}{4}$. Hence find the least of $\mathrm{Im}(\mathbb{Z})$ for points in the region, giving your answer in an exact form.

4. Write the $T_{(r+1)}$ term of the binomial expansion $\left(\sqrt{3}+11^{\frac{1}{5}}\right)^{10}$. Hence, Find the sum of rational terms of the expansion

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5. Evaluate; $\lim_{x \to \frac{\pi}{6}} \left(\frac{12-12\, Cos\left(2x-\frac{\pi}{3}\right)}{(6x-\pi)^2} \right)$

6. The region enclosed by the curves $y = \sqrt{In|x|}, (where\ x > 1, x \in \mathbb{R}), y = 0, x = 2$ and $x = 4$ is rotated about the x axis through 2π radians. Show that the volume of the solid generated is $\boldsymbol{6\pi In(2)}$ cubic units.

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7. Show that the coordinates of any point P with parameter θ on the hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$ can be expressed in the form $(3sec\theta, 6tan\theta)$. Show that the equation of the normal to the given hyperbola at the point with parameter $\theta = \frac{\pi}{6}$ is $x + 4y = 10\sqrt{3}$.

....

8. Let $l = 0$ be a straight line with gradient $m(\neq 0)$ Show that there are two possible positions for $l = 0$, such that the perpendicular distance from O origin to the line $l = 0$ is 1 unit and find the equation of, each of the line $l = 0$.

A rhombus is formed by above mentioned two lines by opposite sides and the two-coordinator axis as diagonals. Show that the arc of the rhombus is $\left|\frac{2(m^2+1)}{m}\right|$

....

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9. A center of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + p = 0$ lying on the line $y = mx + c$ touches the y axis and the intercept made by circle $S = 0$ on the axis is 8 units. Show that,
$g^2(1 - m^2) + 2gmc = 16 + c^2$

10. By using the identity $sin^2\theta + cos^2\theta = 1$ obtain $Cosec^2\theta = 1 + cot^2\theta$ whose $\neq n\pi$, $n \in \mathbb{Z}$. Given that $Cot\theta - Cosec\theta = \frac{5}{4}$ then show that $Cot\theta + Cosec\theta = -\frac{5}{4}$, then show that $Sin\theta = -\frac{40}{41}$

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Part B

※ Answer five questions only.

11. (a) Let $f(x) \equiv x^2 + (2a - x) + (a + 1), x \in \mathbb{R}$ where a is real constant. If $(x + 2a - 1)$ is a factor of $f(x)$. Find the value of a.

Write down $f(x)$ for above a value and obtain the roots of $f(x) = 0$.

Hence write down the quadratic expression $F(x) = f(p - 2x)$, where p is a real constant.

Show that the quadratic equation $F(x) = 0$ has real and distinct roots for all $p \epsilon \mathbb{R}$

Find the quadratic equation $G(x) = 0$ whose roots are the reciprocals of roots of $F(x) = 0$ when $p \neq 0 \; and \; p \neq 3$.

Further, show that both $F(x) = 0$ and $G(x) = 0$ have the same discriminant.

(b) When the polynomial $P(x)$given by $P(x) \equiv x^4 - (1 - \lambda)x^3 + \mu x + 2$ is divided by $x^2 - x - 2$ the remainder is $10(x + 1)$. Find λ and μ.

Show that $(x + 1)$ is a factor of $P(x)$ for above determined λ and μ values.

Express $P(x)$ in the form $P(x) \equiv (x - \alpha)(x^3 - \beta)$ where α and β to be determined.

12. (a) The following table shows some details of a group of persons with their profession.

Profession	Male	Female
Doctor	3	1
Nurse	7	4
Attendant	5	5

A committee of 5 members has to be appointed from this group.
Find the number of different possible committees that can be appointed under each condition.

(i) When there is no any restriction
(ii) All three professions must be participated in the committee and also only for doctors both male and female should be in the committee.
(iii) All the doctors in the group should be participated in the committee.

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(b) Let $f(r) = \frac{2}{(2r-1)^2}$, $r \epsilon \mathbb{Z}^+$

Show that $f(r) - f(r+1) = \frac{16r}{(2r-1)^2(2r+1)^2}$

Write down the rth common term U_r in the infinite series

$\frac{1}{1^2.3^2} + \frac{2}{3^2.5^2} + \frac{3}{5^2.7^2} + \frac{4}{7^2.9^2} + \cdots$

Find V_n and W_{2n} which are defined as $V_n = \sum_{r=1}^{n} u_r$ and $W_{2n} = \sum_{r=1}^{2n} u_r$.

Is $W_{2n} - V_n$ convergent? Justify your answer.

13. (a) Let $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2\alpha & \alpha \\ 0 & 0 \\ -1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$ where α is a real constant.

If $A^T B = 8C$, find $\propto$. Also find $B^T A$ for above $\propto$ value.

Hence show that $A^T B + B^T A$ is a symmetric matrix. Is there exist a 2nd order square matrix P such that $(A^T B)P = I$. Justify your answer. Where I is the 2nd order identity matrix.

(b) Represent the region R on an Argand diagram which satisfies the condition $2 < |Z| \leq 6$ where Z is a complex number. Now let Z_R is the complex number in above region R. Where $Z_R = x + iy$ $(x, y \in \mathbb{R})$

(i) Find Z_0 which is given by $Z_0 = Z_R + \overline{Z_R}$ where $\overline{Z_R}$ is the complex conjugate of Z_R.

(ii) Further show separately the region R' in which, Z_R can exists such that both the complex number Z_R and Z_0 are in the above region R.

(iii) w is the complex number which belongs to the above R' region such that $|w|$ is maximum, $Arg(w)$ is minimum and also in the 1st quadrant. Write down w in $x + iy$ form.

Hence find $w + \overline{w}$ and $w - \overline{w}$ and by using De Moivre's theorem, show that

$(|w + \overline{w}| + i|w - \overline{w}|)^{12} = 12^{12}$.

14. (a) Consider the function $y = f(x) \equiv \frac{3x+p}{(x+q)^2}$,$x \in \mathbb{R}$ where p and q are real constants such that $x \neq -q$. $x = 2$ is a vertical asymptote to the curve $y = f(x)$ and the curve has a stationary point at $x = \frac{4}{3}$. Determine p and q. Show that the first derivative of $y = f(x)$ With relative to x can be expressed as $f'(x) = \frac{4-3x}{(x-2)^3}$; $x \neq 2$

Indicating the intercepts on x axis, intercept on y axis, turning points and asymptotes clearly sketch the curve of $y = f(x)$.

The second derivative of $f(x)$ with relative to x, is given by $f''(x) = \frac{6(x-1)}{(x-2)^4}$, $x \neq 2$

Determine the coordinates of points inflection of the curve $y = f(x)$ and their nature.

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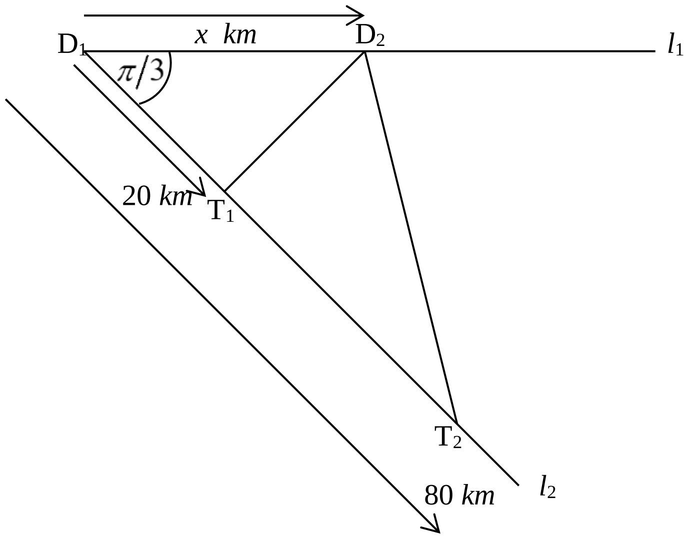
(*b*) In the given figure l_1 and l_2 are the two high tension transmission lines starting from the distribution center D_1 which are in an angle $\frac{\pi}{3}$.

Two distribution transformers T_1 and T_2 are located on the line l_2 at distances $20\ km$ and $80\ km$ respectively, from D_1. It is proposed to established another distribution centre D_2 on line l_1 at a distance x km from D_1 and to join it to T_1 and T_2 using straight transmition lines D_2T_1 and D_2T_2.



Obtain $D_2T_1 = \sqrt{x^2 - 20x + 400}km$ and $D_2T_2 = \sqrt{x^2 - 80x + 6400}km.$

State the range of x in above expressions.

What is the distance from D_1 to the point at which the new distribution center D_2 to be constructed so that it makes the total length of D_2T_1 and D_2T_2 is a minimum.

15. (*a*) For $a \in \mathbb{R}$ where $a > 0$, Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let, $I = \int_0^{\frac{\pi}{2}} \frac{d\theta}{Sin\,\theta\,(Sin^2\,\theta - Cos^2\,\theta)}$ and $J = \int_0^{\frac{\pi}{2}} \frac{d\theta}{Cos\,\theta\,(Sin^2\,\theta - Cos^2\,\theta)}$

Show that $I = -J$. Hence evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{d\theta}{Sin\,\theta\,Cos\,\theta\,(Sin\,\theta - Cos\,\theta)}$

(*b*) Determine the real constants A, B and C such that

$x^2 = (Ax + B)(1 + x)^2 + C(1 + x^2)(1 + x) + D(1 + x^2)$

and obtain the result

$$x^2 = \frac{1}{2}x(1+x)^2 - \frac{1}{2}(1+x^2)(1+x) + \frac{1}{2}(1+x^2)$$

Hence show that,

$$\int \frac{x^2}{(1+x^2)(1+x)^2}\,dx = \frac{1}{2}\left[ln\left|\frac{\lambda\sqrt{1+x^2}}{(1+x)}\right| - \frac{1}{(1+x)}\right]$$

for $x \neq -1$, where λ is a real constant.

(*c*) Using a suitable substitution, evaluate the integral $\int_1^{3^{\frac{1}{4}}} \left(\frac{1}{x^3}\right) tan^{-1}\left(\frac{1}{x^2}\right) dx$

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16. Show that any point P on the straight line $l = 0$ which passes through $A \equiv (2, 1)$ with the gradient m can be expressed parametrically as $P \equiv (2 + t, 1 + mt)$, where t is a parameter.

The rhombus ABCD is entirely in the first quadrant where ABCD is in the counter clockwise sense. Length of a side of the rhombus is 4 units and $A \equiv (2,1)$. Side AB is parallel to ox axis and $B\hat{A}D = \frac{\pi}{3}$.

(i) Using the above parametric representation itself find the coordinates of the vertices B and D of the rhombus ABCD. Hence obtain the coordinates of vertex C.

(ii) Further by using the same parametric representation, find the gradient of the diagonal AC of rhombus and find the equations of the diagonals AC and BD

(iii) Find the equations of circles $S_1 = 0$ and $S_2 = 0$ where sides AB and BC are as diameters of each circle respectively. Are $S_1 \, and \, S_2$ orthogonal. Justify your answer.

(iv) A circle $S_0 = 0$ whose center is on the straight line which passes through the center of rhombus ABCD and parallel to the side AB cuts the circle S_1 orthogonally. Show that S_0 can be expressed as, $S_0 \equiv x^2 + y^2 + 2\lambda x - 2(1 + \sqrt{3})y + (2\sqrt{3} - 11 - 8\lambda) = 0,\ \lambda \in \mathbb{R}$. If the radius of S_0 is $\sqrt{35}$ units then show that there exist such S_0 is circles and find the equations of each circle.

17. (a) Write down $Cos(A + B)$ in terms of $SinA,\ SinB, CosA$ and $CosB$

By selecting A and B properly, obtain the result $Cos[90° + \theta] = -sin\theta.$

Hence show that $Sin110^0 = -Cos200^0$ and $Cos110^0 = Sin20^0$ and deduce that $tan110^0 + cot20^0 = 0$

(b) Prove that $Cos4\theta - Cos2\theta = 8Cos^4\theta - 10Cos^2\theta + 2$

Hence find the values for$Cos\theta$ such that $Cos4\theta = Cos2\theta$

(c) The medians drawn from the vertices A and B of the triangle ABC, to the opposite sides are AD and BE respectively. The lines AD and BE meet at G. Also, by usual notation $a = 4 \, cm \, and \, b = 3 \, cm$. Using the Cosine rule for appropriate triangles, Show that $A\hat{C}B = Cos^{-1}\left(\frac{5}{6}\right)$

(d) Consider the equation, $tan^{-1}(x + 1) + tan^{-1}(x - 2) = tan^{-1}(2)$

Obtain an equation which satisfies x in above equation.

Hence write down suitable solutions for x in above equation.

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